Application of sum rules to heavy baryon masses

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Abstract

Model independent sum rules are applied to recent measurements of heavy c-baryon and b-baryon masses. The sum rules are generally satisfied to the same degree as for the light (u,d,s) baryons.

Recent measurements of the masses of a number of heavy baryons (with c or b quarks) now make it possible to test model independent sum rules that were derived some time ago using fairly minimal assumptions within the quark model.[1] The sum rules depend on standard quark model assumptions, and the additional assumption that the interaction energy of a pair of quarks in a particular spin state does not depend on which baryon the pair of quarks is in ("baryon independence"). This is a weaker assumption than full SU(3) symmetry of the wave function, which would require each individual wave function to be SU(3) symmetrized. Instead, we use wave functions with no SU(3) symmetry as described in Ref.[2]. No assumptions are made about the type of potential, and no internal symmetry beyond baryon independence is assumed. The sum rules allow any amount of breaking in the interactions and individual wave functions, but do rest on baryon independence for each quark-quark interaction energy. A more detailed discussion of the derivation of the sum rules is given in Ref.[1] In a previous paper, we applied an isospin breaking sum rule to the Σ_c charge states.[3] In the present paper we test sum rules that connect baryon states of different isospin, which we characterize as medium strong energy difference sum rules.

Before looking at the heavy baryon masses, we review the application of the model independent sum rules to the light baryons. There, the following sum rules have been derived[2]

$$\tfrac{1}{3}(\Omega^{*-} - \Delta^{++}) \;=\; \Xi^{*0} - \Sigma^{*+} \;=\; \Xi^0 - \Sigma^+, \tag{1}$$

$$\quad (147 \pm 1) \qquad\qquad (149 \pm 1) \qquad\qquad (125)$$

$$2\overline{N} + 2\overline{\Xi} - 3\Lambda - \overline{\Sigma} \;=\; \Omega^* + \overline{\Delta} - \overline{\Sigma^*} - \overline{\Xi^*}. \tag{2}$$

$$\quad (-26) \qquad\qquad\qquad (-14 \pm 2)$$

The baryon symbol has been used for its mass, and, except for the Δ, a star indicates spin $\tfrac{3}{2}$. A bar over the symbol represents an average over the particular isospin multiplet.

Where specific charges are indicated, these could be changed using the isospin breaking sum rules in Ref.[1]. The experimental value in MeV for each sum is given below each equation.

The Gell-Mann Okubo formula and equal spacing in the decuplet, which would follow if $SU(3)$ symmetry were broken only by a small octet component, correspond to each side of Eq. (2) being zero. The deviations of each side in Eq. (2) from zero indicate that the light baryon interactions do not satisfy that assumption (which is not required for the sum rules), with breaking of the order of 10 MeV. The deviation between the two sides in Eq. (2) and by the $\Xi^0 - \Sigma^+$ term in Eq. (1) indicates that the light baryon wave functions also violate our baryon independence assumption to the extent of about 10-20 MeV in the masses. We note that the breaking of baryon independence occurs for the two cases where baryons of different spin are compared. Similar breaking should be expected in the heavy baryon sum rules, even if the heavy spin-spin interactions are more $SU(3)$ and $SU(6)$ symmetric.

For extension to heavy baryons, it is convenient to use the equalities in Eq. (1) to replace Eq. (2) by

$$(\Delta^+ - p) \;=\; (\Sigma^{*0} - \Sigma^0) + \tfrac{3}{2}(\Sigma^0 - \Lambda^0) \;=\; (\overline{\Xi^*} - \overline{\Xi}) + \tfrac{3}{2}(\Sigma^0 - \Lambda^0). \tag{3}$$

$$\quad (297) \qquad\qquad (307) \qquad\qquad\qquad (330)$$

Equation (3) shows a spread of ~ 30 MeV corresponding to breaking of that order of baryon independence in the light baryons. This demonstrates the ambiguity that arises when the sum rules are extended to heavy baryons. We will try to minimize this ambiguity by choosing the most suitable light

Baryon	Mass (MeV)	Reference
Σ_c^{++}	$\Lambda_c^+ + 168.0 \pm 0.3$	[4]
Σ_c^+	$\Lambda_c^+ + 168.7 \pm 0.4$	[4]
Σ_c^{*++}	$\Lambda_c^+ + 245 \pm 7$	[5]
$\Xi_c'^+$	2563 ± 15	[6]
Ξ_c^{*0}	2643.3 ± 2.2	[7]
Ω_c^0	2700 ± 3	[8]
Σ_b^-	$\Lambda_b^0 + 173 \pm 9$	[9]
Σ_b^{*-}	$\Lambda_b^0 + 229 \pm 6$	[9]

Table 1: Heavy baryon masses used in the sum rules.

baryon combination to compare to heavy baryons.

In table I, we list the measured heavy baryon masses that will be used in the sum rules. We indicate the expected baryon assignments in table I. The $\Xi_c'^+$ is the spin $\frac{1}{2}$ usc baryon having the u-s quarks in a spin 1 state. We extend Eq. (3) to charmed baryons by changing the s-quark into a c-quark. This leads to[10]

$$(\Sigma^{*0} - \Lambda^0) + \tfrac{1}{2}(\Sigma^0 - \Lambda^0) = (\Sigma_c^{*+} - \Lambda_c^+) + \tfrac{1}{2}(\Sigma_c^+ - \Lambda_c^+) \tag{4}$$
$$(307) \qquad\qquad\qquad (330 \pm 7)$$

We have used the light Sigma baryons for the left hand side of Eq. (4). Use of other combinations could change the left hand side, as indicated in Eq. (3), but the Sigma combinaton is the most reasonable since they are most similar to their charmed counterparts. Equation (4) is written in terms of differences from the Λ mass which is how the Σ_c and Σ_c^* masses are measured. We have had to use the measured Σ_c^{*++} mass for the Σ_c^{*+} mass in Eq.(4), but that difference is probably small.

Changing the c-quark in any c-baryon sum rule to a b-quark leads to the corresponding sum rule for b-baryons. Applying this to Eq. (4) leads to

$$(\Sigma^{*0} - \Lambda^0) + \tfrac{1}{2}(\Sigma^0 - \Lambda^0) = (\Sigma_b^{*0} - \Lambda_b^0) + \tfrac{1}{2}(\Sigma_b^0 - \Lambda_b^0) \tag{5}$$
$$(307) \qquad\qquad\qquad (316 \pm 10)$$

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The sum rules in Eqs. (4) and (5) are satisfied to about the same extent as the light baryon sum rules.

In Ref.[3] we used a sum rule to predict the Ξ'_c mass which has now been measured. This permits a test of the sum rule, which we write here as

$$\Sigma^+ + \Omega^{*-} - \Xi^0 - \Xi^{*0} \;=\; \Sigma_c^{++} + \Omega_c^0 - 2\Xi_c'^+. \tag{6}$$
$$(15) \qquad\qquad\qquad\qquad (27 \pm 30)$$

Again, we have used the combination of light baryon masses most similar to the corresponding charmed baryons. However, the left hand side of Eq. (6) could be made to vary between -3 and +27 MeV, by using Eq. (1) to substitute other light baryon combinations .

The spin $\frac{3}{2}$ counterpart of Eq. (6) can be used to predict the as yet unmeasured Ω_c^{*0} mass

$$\Omega_c^{*0} = \Omega_c^0 + 2(\Xi_c^{*+} - \Xi_c'^+) - (\Sigma_c^{*++} - \Sigma_c^{++}) = 2783 \pm 30, \tag{7}$$

where we have used the most similar c-baryon combination rather than using any light baryons. This increases the error on the prediction, but is the more reasonable choice.

We see that, especially when the most reasonable combination of light baryons is taken, the medium strong energy difference sum rules are satisfied at least as well for the heavy baryons as for the light quark baryons. However the situation is not as nice for the isospin breaking mass differences in the case of the Σ_c. In Ref.[3] we showed that the Σ_c sum rule is violated by three standard deviations, while the corresponding light baryon sum rule is satisfied. Since sum rules in disagreement are of more concern than those which are satisfied, resolving the Σ_c mass differences is of prime importance.

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[10] Equation (4) is a combination of several equations in Ref. [1].